A Planar Metamaterial Co-Directional Coupler That Couples Power Backwards

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Abstract — **A metamaterial coupler made of a regular microstrip line and a negative-refractive-index (NRI) line is examined. Peculiar coupling effects are observed in which co-directional traveling waves result in contra-directional Poynting vectors on the lines thus leading to power being coupled backwards. This coupler demonstrates better performance in terms of coupled power and port isolation, but at a slightly reduced bandwidth, when compared to a commensurate regular microstrip coupled-line coupler.**

I. Introduction

The feasibility of media exhibiting simultaneously negative permeability and permittivity, thus leading to a negative refractive index, was predicted by Veselago in the sixties [1]. Veselago termed these media "left handed" because the **E**, **H** and **k** vectors form a left-handed triplet and thus support backward propagating waves. The first experimental demonstration of negative refraction was presented in [2] based on such a "left-handed" or negative refractive index (NRI) medium (metamaterial) constructed using a periodic arrangement of printed wires and split-ring resonators. Last year in this conference, Iyer and Eleftheriades introduced a low profile, broadband 2-D NRI metamaterial that is based on periodically loading a host microstrip transmission-line (TL) medium with series capacitors and shunt inductors. Using this approach, experimental demonstration of focusing from a planar dielectric/NRI medium was shown [3,4].

Despite the seeming maturity of printed couplers, new coupling phenomena in planar structures are still being discovered [5]. In this paper, we present a peculiar coupled-line NRI metamaterial coupler in which the waves travel forward (i.e. the phase velocities point away from the source) but the power is coupled backwards. This coupler consists of a regular microstrip line and a metamaterial negative refractive index line (MS/NRI coupler). Note that since the refractive index is defined as the ratio of the speed of light to the phase velocity (positive or negative), a NRI line (1-D) can be readily defined.

Coupled-line couplers rely on proximity interaction of fields between transmission lines to transfer power between them. In co-directional coupled-line couplers, the direction of the propagation constants on the two lines is the same. When such couplers are constructed using either two regular or two "left-handed" lines [6], the underlying propagation phenomenon is fundamentally similar to that on a corresponding isolated line of the same type. Indeed in such co-directional couplers consisting of two lines of the same type, the power also couples co-directionally and appears at the far end of the coupler (i.e. away from the source). Peculiar features arise when two different types of lines are used. Indeed it is hereby shown that a MS/NRI coupler excites co-directional forward traveling waves but delivers power backwards as a result of the power flow being contra directional. Fig. 1(a) shows a loaded transmission-line based realization of a NRI line [3]. A schematic of the examined MS/NRI coupler is shown in Fig. 1(b).

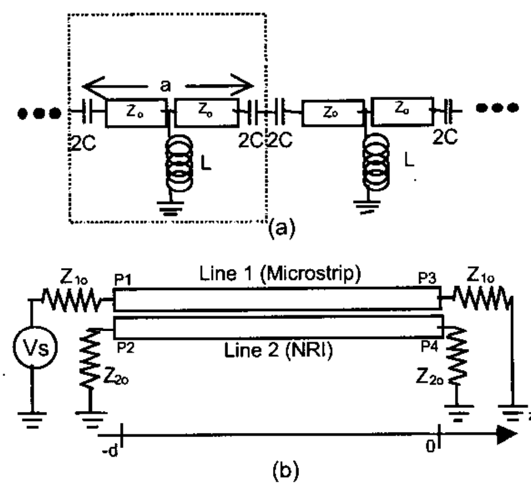

Fig. 1. (a) Loaded-transmission-line based Negative Refractive Index (NRI) line (b) MS/NRI line coupler

II. Theory of Operation

The operation of coupled-line forward couplers can be understood in terms of constructive and destructive interference between the two independent modes that can be excited. This, combined with matching terminations to avoid reflections, results in power transfer from the excited to the coupled line.

In NRI media, electromagnetic waves with phase fronts traveling forward ($\exp(-j\beta_z z)$ in Fig.1) carry power backwards [1], [3], [7, 8]. Hence if the MS line is excited at port 1 in Fig. 1(b) (assuming weak coupling), then due to phase matching conditions, the power on the NRI line is expected to couple backwards (i.e. at port #2). It should be pointed out that this situation is analogous to the phenomenon of backward-wave leakage from a NRI medium to a regular medium (e.g. air). Such backward radiation from a NRI line, supporting a fundamental spatial harmonic, has been demonstrated before in [7, 8]. Therefore anticipating backward power coupling by forward waves in the proposed coupler is reasonable.

A. Assumptions for simplifying analysis

A system of coupled-mode differential equations can be written describing the voltages and currents on the lines [9]. The resulting eigenvalue equation gives the propagation constants 'β' for the two modes (denoted by subscripts 'c' and 'π') of the system.

Some assumptions to derive an expression relating the input power at port 1 to the coupled power at port 2, are as follows. First, it is assumed that the lines are lossless and hence the propagation constants are purely imaginary. Second, 'weak' coupling between the lines is assumed so that the propagation constants β_c and β_π are close to the propagation constants β_1 and β_2 of the isolated lines. Lastly it is assumed that the lines are properly terminated so that there are no reflections. Reflections are minimized when the two possible propagation constants are similar in magnitude; this is crucial for improving the amount of coupled power. There is a tradeoff here – it will be shown that the closer β_c is to β_π, the longer the required line length for optimal coupling. Hence a designer has to optimize the coupler trying to eliminate reflections and at the same time keep the optimal/coherence length of the line to a minimum. Fortunately this turns out not to be a severe constraint.

B. Estimation of the coupled power

From the given assumptions and the fact that the lines are properly terminated it is evident that the waves for each mode in the lines travel in only one direction (as there are no reflections). Moreover $\beta_c \approx \beta_1 > 0$ and $\beta_\pi \approx \beta_2 < 0$ due to the choice of materials and weak coupling.

One has to be careful in deciding the signs for the line current expressions. In regular transmission lines, currents with forward traveling phase fronts are associated with positive signs and backward traveling phase fronts are associated with negative signs. This ensures that the Poynting vector is parallel with the phase flow direction. But this is reversed in the case of NRI lines where the Poynting vector is antiparallel to phase flow. Keeping this in mind along with the signs of β_1 and β_2, and denoting Z_{1o} and Z_{2o} as the line impedances, the following expressions for line voltages and currents can be written:

$$V_1 = V_C e^{-j\beta_1 z} + V_\pi e^{j\beta_2 z} \tag{1}$$

$$V_2 = R_c V_C e^{-j\beta_1 z} + R_\pi V_\pi e^{j\beta_2 z} \tag{2}$$

$$I_1 = (V_C / Z_{1o}) e^{-j\beta_1 z} + (V_\pi / Z_{1o}) e^{j\beta_2 z} \tag{3}$$

$$I_2 = -(R_c V_C / Z_{2o}) e^{-j\beta_1 z} - (R_\pi V_\pi / Z_{2o}) e^{j\beta_2 z} \tag{4}$$

where R_c and R_π are the ratios of the voltage in line 2 to line 1 of the c-mode and π-mode respectively.

Applying boundary conditions at port 1 (P1) and port 4 (P4) in Fig. 1(b) one can solve for V_c and V_π in terms of V_s, R_c, R_π and the propagation constants. Substituting back into equations (1) and (2) yields the following expression for the line voltage profiles:

$$V_1(z) = \frac{V_s}{2} \frac{R_\pi e^{-j\beta_1 z} - R_c e^{j\beta_2 z}}{R_\pi e^{j\beta_1 d} - R_c e^{-j\beta_2 d}} \tag{5}$$

$$V_2(z) = \frac{V_s}{2} \frac{R_\pi R_c (e^{-j\beta_1 z} - e^{j\beta_2 z})}{R_\pi e^{j\beta_1 d} - R_c e^{-j\beta_2 d}} \tag{6}$$

From equation (6), it is indeed confirmed that port 4 (p4) is isolated (line 2, z=0) and that port 2 (p2) is the coupled port. Moreover, expressions for power delivered to ports 2 and 3 can be obtained from (5) and (6):

$$P_2 = \frac{1}{Z_{2o}} \left(\frac{V_s}{2}\right)^2 \frac{(R_C R_\pi)^2 (1 - \cos((\beta_1 + \beta_2) d))}{(R_c^2 + R_\pi^2 - 2 R_c R_\pi \cos((\beta_1 + \beta_2) d))} \tag{7}$$

$$P_3 = \frac{1}{2 Z_{1o}} \left(\frac{V_s}{2}\right)^2 \frac{(R_C - R_\pi)^2}{(R_c^2 + R_\pi^2 - 2 R_c R_\pi \cos((\beta_1 + \beta_2) d))} \tag{8}$$

Equation (8) yields the coherence length at which maximum power transfer to port 2 takes place:

$$d_{max.power} = \pi / |\beta_1 + \beta_2| \tag{9}$$

III. Simulation Results

Presented simulation results are from Agilent HP-ADS®. The coupler is designed on a 50-mil substrate of dielectric constant 4.5. Microstrip transmission line segments of width 2.385mm are used of isolated impedance 50Ω.

A. An optimized short-length MS/NRI coupler

Simulation is carried out at 2.6GHz with unit cells of length 3mm and load elements L=13.2nH and C=1pF (refer to Fig. 1(b)). A separation of 0.4mm is used between the lines. The impedances Z_{1o} and Z_{2o} are 48Ω and 46Ω respectively.

From band calculations for the periodic structure representing the NRI line [10] and HP-ADS simulations of the RHM microstrip line, the propagation constants for the two modes differ enough at 2.6GHz to give a reasonable coherence length (equation 9).

Discrete spatial voltage Fourier transforms of the two coupler lines are taken with 500 samples each and at a sampling interval of 3mm to determine the coupled mode propagation constants and the mode voltage ratios R_c and R_π. By observing the peaks in the spectrum it is determined that β_1=101m^{-1} and β_2=-306m^{-1}. These values are very similar to the isolated line propagation constants. Ratios of the Fourier coefficients between line 2 spectrum and line 1 spectrum at points β_1 and β_2 give $R_c \approx -0.124$ and $R_\pi \approx -7.5$ respectively.

From equation (9), the expected coherence length is about 15.3mm. Moreover, the power input at port 1 can be approximated by $V_s^2/(8Z_{1o})$ and along with equation (8) the scattering parameter S_{12} at the design frequency, is estimated to be -12.1dB. Recalling that the cell size is 3 mm, simulation results show a coherence length of 5 cells or 15mm which is close to the predicted value (Fig. 2). Moreover the S_{12} parameter is about −12dB as predicted.

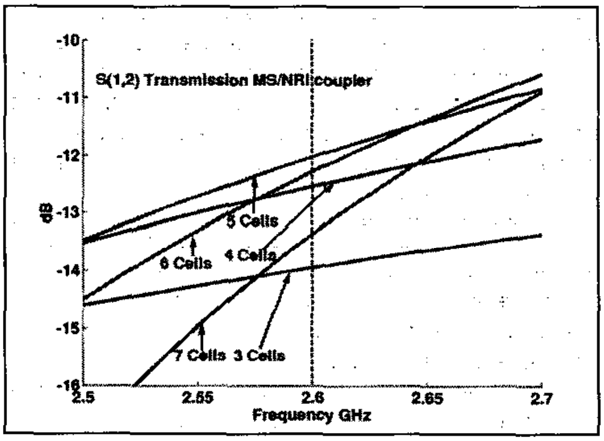

Fig. 2. Scattering parameter S(1,2) against frequency for various coupler lengths measured in terms of unit cells.

B. Comparison of a MS/NRI coupler and a regular microstrip coupled- line coupler of the same length

In order to compare a regular microstrip coupler [10] to the new MS/NRI coupler, a design frequency of 2.9GHz is chosen. This makes the magnitudes of β_1 and β_2 very close and similar to the propagation constant of a conventional (contra-directional) microstrip coupler constructed with similar geometry but without any loading [10]. The latter step is necessary to avoid neglecting the fact that it is possible to shorten the optimum length of a regular coupler by appropriately loading it to slow down the propagating waves.

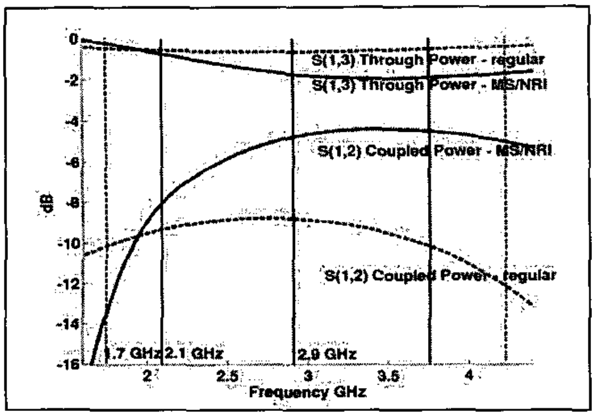

Fig. 3. S(1,2) and S(1,3) for MS/NRI coupler (in solid line) & regular coupled-line MS/MS coupler (in dotted line).

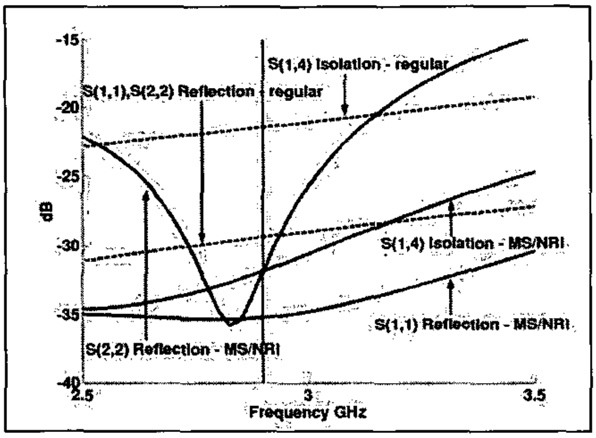

Fig. 4. S(1,1), S(2,2) and S(1,4) for MS/NRI coupler (in solid line) & regular MS/MS microstrip coupler (in dotted line).

For simulation carried out at 2.9GHz, the unit cell for the NRI line is 5 mm long with lumped elements L=13.2 nH and C=1pF. The microstrip line propagation constant at this frequency is $112.7m^{-1}(\approx\beta_1)$. From band calculations of the NRI line, the isolated line propagation constant is $-101.0m^{-1}$ ($\approx\beta_2$). Separation between the lines is 0.1mm and this setup requires matching impedances Z_{1o} and Z_{2o} of 44 Ω and 59 Ω respectively.

Figures 3 and 4 compare the coupling characteristics of a regular MS coupler of length 15cm (λ/4, i.e. optimum length) with a MS/NRI coupler (non optimum length) of the same length. As shown, the latter performs better (compared to the regular coupler) in terms of coupled power and port isolation but suffers from slightly reduced bandwidth. At the design frequency of 2.9 GHz, the coupled power at port 2 of the MS/NRI coupler is about 4dB higher than in the regular coupler (see Fig. 3). But the –3dB bandwidth for the former is around 1.6GHz and 2.4GHz for the latter. The port isolation at the design frequency is again better for the MS/NRI coupler (see Fig. 4). Comparison of the power coupled to port 2 and port 4 indicates that this forward-wave, co-directional coupler behaves (in terms of power flow) as a contra-directional coupler (Fig. 3 and 4). Moreover the power leakage into the isolated port is minimized.

As β's are almost equal for both modes, the coherence/optimized length is very large for the MS/NRI coupler (equation 9). Indeed, for this coupler, it is possible to make the transmission parameter S_{12} very close to 0dB by increasing its length to 12 cells.

Lastly from simulations it can be verified, by observing the decreasing phase as one moves away from the source end in the coupler lines (Fig. 5), that there is a dominating presence of phase fronts traveling forwards on both lines as assumed in the analysis.

IV. Conclusion

Peculiar co-directional couplers can be made using one microstrip and one NRI line. Such devices excite co-directionally traveling waves on the lines but the Poynting vectors couple contra-directionally, resulting to the power being delivered backwards. It is shown that such a MS/NRI coupler has superior performance in terms of coupled power and port isolation when compared to a traditional coupled-line (contra-directional) microstrip coupler of the same length (λ/4). Derived analytical expressions describing the coupler agree well with microwave circuit simulations.

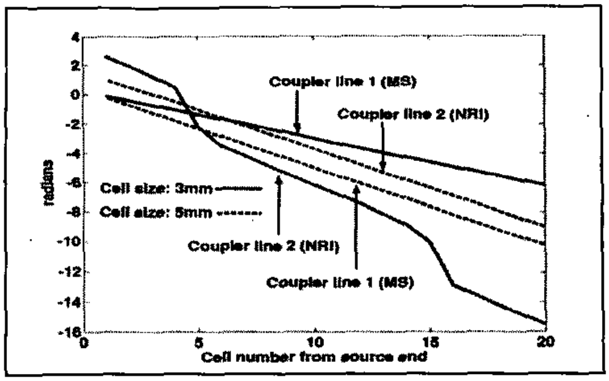

Fig. 5. Voltage phase as a function of distance from source end

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